Equation Decoupling—A New Approach to the Aerodynamic Identification of Unstable Aircraft

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This paper presents the equation decoupling technique as a new approach to the parameter estimation of both aerodynamically stable and highly unstable aircraft via the output error method. It is shown that this technique not only eliminates the numerical difficulties of integrating the unstable system and sensitivity equations, but also leads to a considerable reduction of CPU time consumption. Simulated data of an advanced, aerodynamically highly unstable combat aircraft study, obtained from a nonlinear six degree-of-freedom simulation, are used to illustrate the technique. Nondimensional derivatives are estimated using both linear and nonlinear kinematic models of the aircraft, and a comparison of the results with the corresponding reference data from the aerodynamic data set is performed. It is concluded that the equation decoupling technique is an efficient tool for parameter estimation purposes in flight test environments for both aerodynamically stable and highly unstable aircraft.

Nomenclature

F, G	= system and control matrix
I	= identity matrix
M	= Mach number
m	= pitch moment
$\boldsymbol{n}$	= measurement noise vector
nz	= load factor z direction (body axis)
q	= pitch rate
T, N	= tangential and normal force, respectively
$\boldsymbol{u}$	= control vector
V	= true airspeed
$\boldsymbol{x}$	= aircraft state vector
$x'(t)$	= $\mathrm{d}/\mathrm{d}t[x(t)]$
y	= measured aircraft state vector
α	= angle of incidence
β	= angle of sideslip
ε	= leading-edge control surface
ηcan	= canard control surface
$\eta 1-\eta 4$	= trailing-edge control surfaces 1–4
Θ	= pitch attitude
$\boldsymbol{\pi}$	= vector of aerodynamic parameters

Introduction

PRESENT and future combat aircraft are, at least in parts of their flight envelopes, aerodynamically highly unstable. The parameter estimation of aerodynamically highly unstable aircraft showing characteristic times to double amplitude as low as 0.2 s via the output error method causes immense numerical difficulties in integrating the unstable system and sensitivity equations, which makes the direct application of this method usually impossible.

On the other hand, the output error method is, because of its comparative simplicity and universal applicability, the most widely used method for aerodynamic parameter estimation. In fact, it appears to be the only practicable method in flight test environments where many flight conditions and maneuvers have to be analyzed in a short time scale, usually on an interflight routine basis. Although filter error methods are theoretically attractive, they are used rarely for practical aerodynamic parameter estimation purposes.[1]

It is, therefore, highly desirable to extend the range of applicability of the output error method into the area of highly unstable aircraft. The equation decoupling technique, presented in the following, is a method to accomplish this task.

In this paper, first the method of analysis is described and the difficulties of direct application of the output error method to unstable aircraft are shown. Then, the concept of equation decoupling is explained. Finally, it is applied to simulated data of an advanced, aerodynamically highly unstable combat aircraft study, obtained from a nonlinear six degree-of-freedom simulation. A comparison of the results with the corresponding reference data from the aerodynamic data set is performed.

Method of Analysis and Data Used

The aircraft motion is given by a nonlinear six degree-of-freedom simulator, which uses a nonlinear aerodynamic data set that contains the complete aerodynamic force and moment coefficients as functions of α, β, Mach number, and control surface positions. The force and moment coefficients are linearized at the trim point, yielding to the reference aerodynamic derivatives with which the results that are obtained from the parameter estimation program[2] are compared. Offsets of 50–125%, related to the reference values, are added to some of these reference derivatives, which are then used as starting values for the estimation program.

It is stated explicitly that the aerodynamic linearization introduces a model error and the estimated derivatives will have some (small) bias relative to the reference data.

A representative point within the flight envelope was chosen, and a 3211 input was injected at the pitch stick. Figure 1 shows the stick movement together with the resulting canard and trailing-edge control surface movements. Because of the high correlation between the canard and the trailing-edge movements, only canard control power derivatives were estimated, keeping the control power derivatives of the trailing edge fixed at the reference value. The time histories of the resulting longitudinal aircraft motion is shown in Fig. 2. Simulated measurement noise is added to these time histories in order to give more realistic conditions for the investigation.

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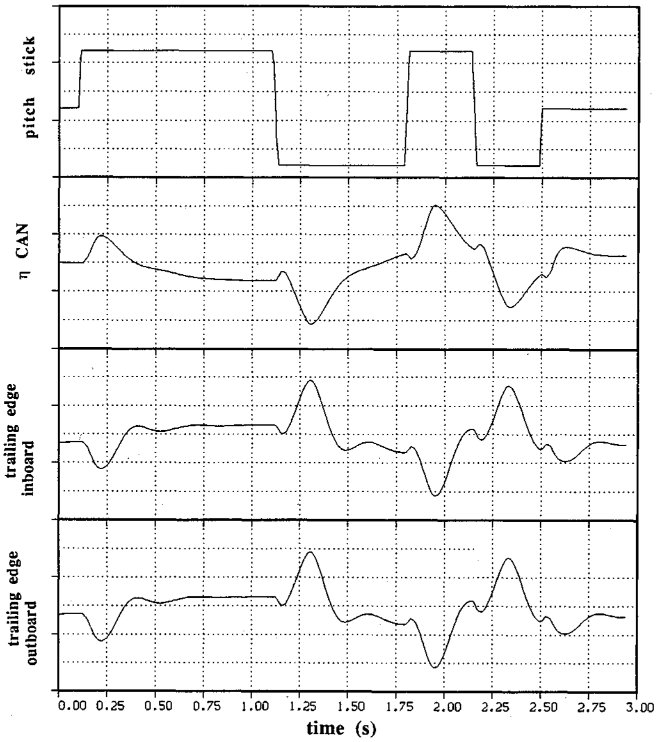

Fig. 1 Pitch stick and control surface deflections (inputs).

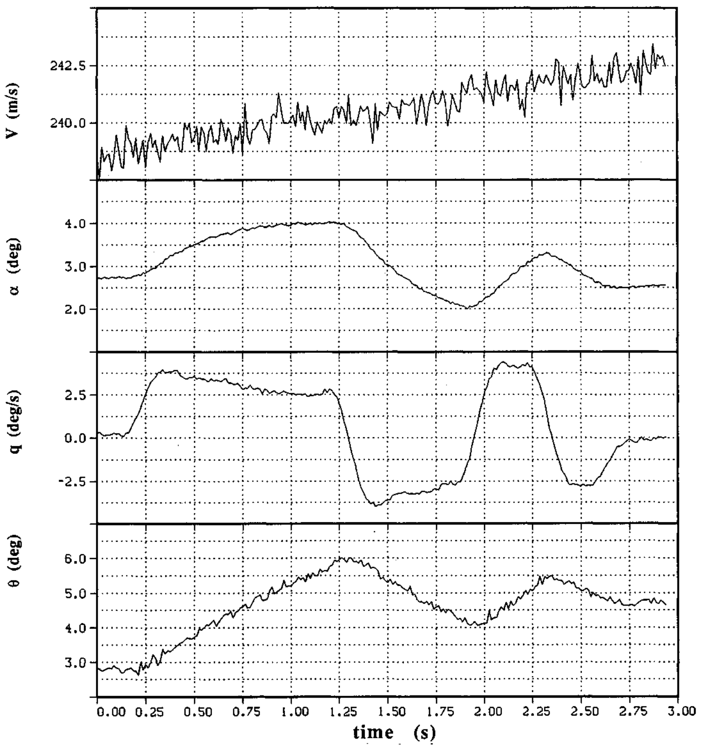

Fig. 2 Simulated A/C motion, contaminated with white noise.

These noise-contaminated time histories serve as measured A/C data for the parameter estimation program in this investigation.

Due to classification, only relative, rather than absolute, quantities, are presented in this paper.

Application of the Output Error Method

In order to avoid the necessity of modeling the flight control system (FCS) for aerodynamic parameter estimation purposes, it is natural to use the control surface deflections as inputs to the mathematical model of the aircraft. Therefore, in the case of aerodynamically unstable aircraft, an unstable system of differential equations is to be integrated. This integration, performed in highly unstable systems as in the sample case, usually ends up in machine overflows and program terminations due to three reasons, which are at least partly present in almost every flight test maneuver that will be analyzed.[3,4] First, due to measurement errors and noise and the finite computer word length in the processing of the flight data, there are errors in the initial conditions of the aircraft state variables. Second, the starting values for the aerodynamic derivatives used in the model are usually wrong, and some of these derivatives are kept fixed at their starting values during the estimation process. Third, model errors in both the kinematic and aerodynamic model are present, and process noise might also play a part to some extent in the maneuver to be analyzed.

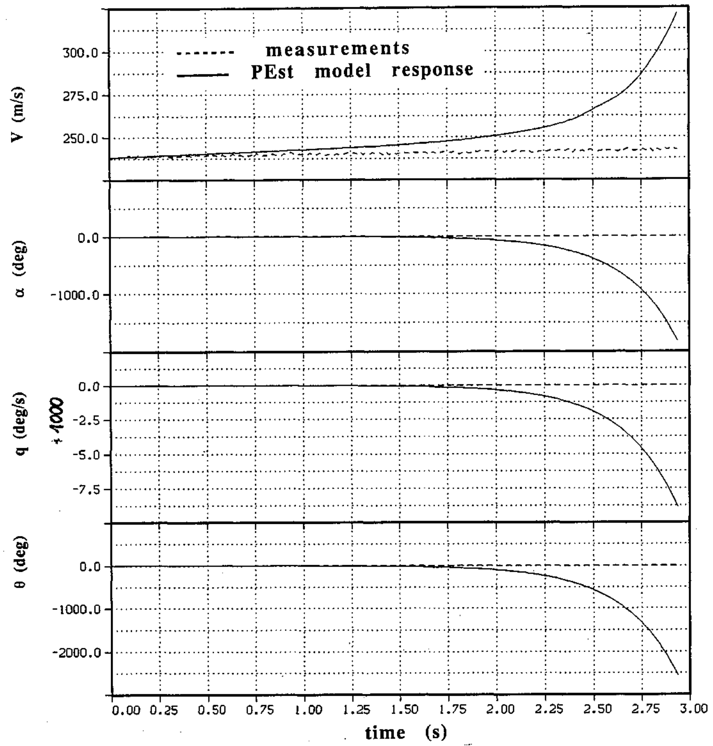

Fig. 3 Comparison of measurements and PEst model, using reference derivatives in estimation model.

For the example shown in Fig. 2, the difference between measurements and the response of the model used in the parameter estimation program is shown in Fig. 3. It can be seen that the system is unstable with a time to double of about 0.2 s; after 2.9 s, the pitch rate is already about −8700 deg/s. The reference aerodynamic derivatives were used as starting values, and no parameter estimation was possible due to machine overflows and resulting program termination.

It is concluded that the application of the output error method to highly unstable systems is, in general, not possible.

Equation Decoupling Technique

The equation decoupling technique[5] is equally usable in both linear and nonlinear systems, but in order to keep it simple, it is explained here as a linear system with measurement matrix equal to I.

The linear system is then given by

$$x'(t) = F(\pi)x(t) + G(\pi)u(t) \tag{1}$$

$$y(t) = x(t) + n(t) \tag{2a}$$

The basic idea of the equation decoupling technique is the introduction of two $(n \times n)$ matrices KO and KOI, whose elements underlie the following restrictions:

$$KO(i,j) = 0,1 \qquad i,j = 1,...,n \tag{2b}$$

$$KO(i,i) = 1 \qquad i = 1,...,n \tag{2c}$$

$$\text{if } \{KO(i,j) = KO(i,k) = 1\} \text{ then } KO(j,k) = 1 \quad i,j,k = 1,...,n \tag{2d}$$

$$KOI(i,j) = 1 - KO(i,j) \qquad i,j = 1,...,n \tag{2e}$$

The system of differential equations [Eq. (1)] is then changed into

$$x'(t) = F(\boldsymbol{\pi})[KOx(t) + KOIy(t)] + G(\boldsymbol{\pi})u(t) \tag{3}$$

Consider as an extreme case the completely decoupled system ($KO = 1$) of order $n = 4$ with two inputs:

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \pi_{11} & 0 & 0 & 0 \\ 0 & \pi_{22} & 0 & 0 \\ 0 & 0 & \pi_{33} & 0 \\ 0 & 0 & 0 & \pi_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & \pi_{12} & \pi_{13} & \pi_{14} \\ \pi_{21} & 0 & \pi_{23} & \pi_{24} \\ \pi_{31} & \pi_{32} & 0 & \pi_{34} \\ \pi_{41} & \pi_{42} & \pi_{43} & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} + \begin{bmatrix} \pi_{15} & \pi_{16} \\ \pi_{25} & \pi_{26} \\ \pi_{35} & \pi_{36} \\ \pi_{45} & \pi_{46} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{4}$$

It can be seen that the only calculated (integrated) variable entering the differential equation for $d/dt\{x_\nu(t)\}$ is $x_\nu(t)$ ($\nu = 1,...,4$), and all other variables are measured data. As a consequence, each differential equation could be integrated independently of the others, and, therefore, the equations are completely decoupled.

The decoupling of the equations may change the unstable system into a decoupled stable system. This is illustrated in the following numerical example of a linear, unstable system of order $n = 4$ [see also Eq. (1)]. Only the system matrix F is shown:

$$F = \begin{bmatrix} -0.2 & -5 & 0 & 10 \\ 0 & -1 & 1 & -0.2 \\ 0 & 6 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Now, following the rules in Eqs. (2b–2e), the third equation is decoupled from the system using the matrices KO and KOI [KOI follows from Eq. (2e)]:

$$KO = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

The resulting system matrix of the decoupled system (see also Eq. 3) is then

$$F^* = \begin{bmatrix} -0.2 & -5 & 0 & 10 \\ 0 & -1 & 0 & -0.2 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

As can be seen, e.g., by the Hurwitz criterion, the decoupled system is now asymptotically stable, and, therefore, the equation decoupling technique can change the stability properties of the system. Even in the case that the decoupled system is still unstable, which is possible in principle but was never the case up to now, the introduction of the measured A/C state variables into the system via the matrix KOI prevents the simulated state variables from rapid divergence.[5]

The stabilizing effect of the decoupling technique can be seen in Fig. 4, where the same maneuver as in Fig. 3 is simulated with the parameter estimation model, but now the system is completely decoupled. There is no divergent behavior anymore, and the residual mismatch between the time histories is purely a consequence of the aerodynamic linearization errors.

Between the completely decoupled ($KO = I$) and coupled ($KOI = 0$) cases, several partially coupled systems are meaningful.

With respect to parameter estimation, it was found that the decoupling of the equations leads to an enlarged radius of convergence for the parameter estimates, and the speed of convergence is improved (usually 3–4 iterations are necessary) in comparison with the completely coupled system.

Another advantage of the decoupling is the reduction of model complexity of the sensitivity functions. To see this, suppose that only the parameter π_{12} is to be estimated. In that case, the sensitivity equations for the completely coupled system also give a coupled system of differential equations:

$$\frac{d}{dt}\frac{\partial}{\partial \pi_{12}}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} \\ \pi_{21} & \pi_{22} & \pi_{23} & \pi_{24} \\ \pi_{31} & \pi_{32} & \pi_{33} & \pi_{34} \\ \pi_{41} & \pi_{42} & \pi_{43} & \pi_{44} \end{bmatrix} \frac{\partial}{\partial \pi_{12}}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} x_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

with

$$dx_\nu(t=0)/d\pi_{12} = 0, \qquad \nu = 1,\dots,4$$

In the completely decoupled case, this is reduced to only one equation:

$$\frac{d}{dt}\frac{\partial}{\partial \pi_{12}}[x_1] = [\pi_{11}]\frac{\partial}{\partial \pi_{12}}[x_1] + y_2 \tag{6}$$

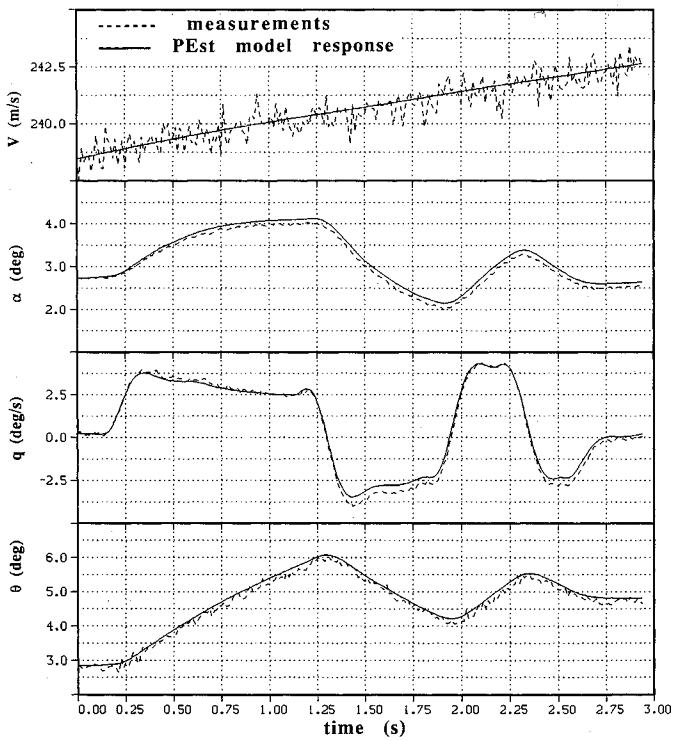

Fig. 4 Comparison of measurements and PEst model, using reference derivatives in estimation model (equations completely decoupled).

with

$$\frac{\mathrm{d}x_1(t=0)}{\mathrm{d}\pi_{12}} = 0$$

Due to this reduction of model complexity, the CPU time consumption is also reduced to about 50% as compared with the completely coupled case.

In practice, the estimation is started using completely decoupled equations and switched to partially or completely coupled equations after several iterations. For stable systems, this procedure makes no problems. In a highly unstable system, however, it is usually not possible to arrive at the completely coupled system. For the example described in this paper, it is necessary to decouple at least the q equation from the system in order to make the parameter estimation possible.

Results and Discussion

The output error method, including the equation decoupling technique, is now applied to the maneuver shown in Fig. 2. The time slice consists of 236 samples and the sampling period is 0.0125 s. For illustration purposes, the starting values of the derivatives to be estimated are set off from the corresponding reference data by amounts of 50–125% relative to the reference values, and nine iterations were run with the parameter estimation program. Both linear and nonlinear kinematic models were used, but only the results obtained by the linear kinematic model are discussed because the corresponding data of the nonlinear kinematic model are identical

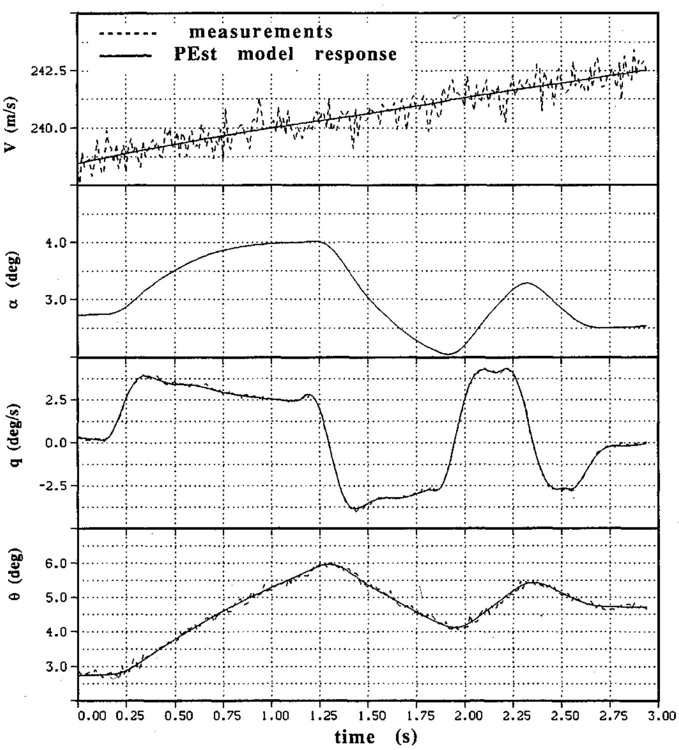

Fig. 5 Matching, using estimated derivatives (results) in estimation model (equations completely decoupled).

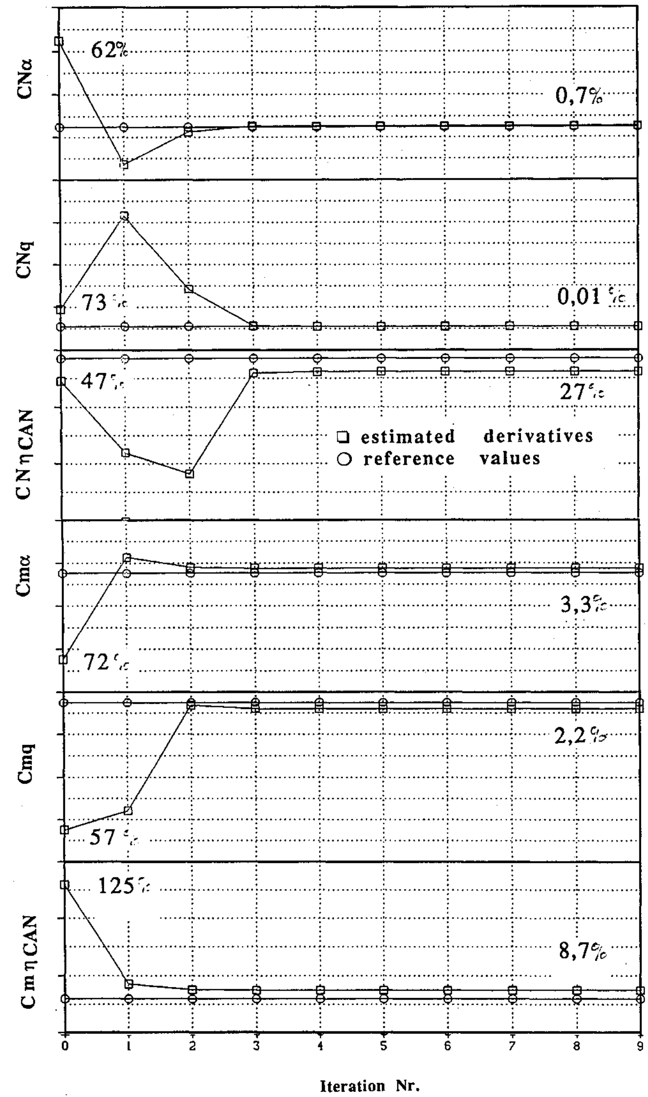

Fig. 6 Convergence behavior of the estimated derivatives.

Table 1 Derivatives used in the model and relative offsets of starting and final values to reference values

		Offsets, %			Offsets, %	
		Start	Final		Start	Final
$CT\alpha$	$CN\alpha$[a]	62	0.7	$Cm\alpha$[a]	72	3.3
CTM	CNq[a]	73	0.01	Cmq[a]	57	2.2
$CT\eta$can	$CNnz$			$Cmnz$		
$CT\eta 1$	CNqdot			Cmqdot		
$CT\eta 2$	CNM			CmM		
$CT\eta 3$	$CN\alpha$dot			$Cm\alpha$dot		
$CT\eta 4$	$CN\eta$can[a]	47	27	$Cm\eta$can[a]	125	8.7
$CT\varepsilon$	$CN\eta 1$			$Cm\eta 1$		
	$CN\eta 2$			$Cm\eta 2$		
	$CN\eta 3$			$Cm\eta 3$		
	$CN\eta 4$			$Cm\eta 4$		
	$CN\varepsilon$			$Cm\varepsilon$		

[a]Estimated derivatives.

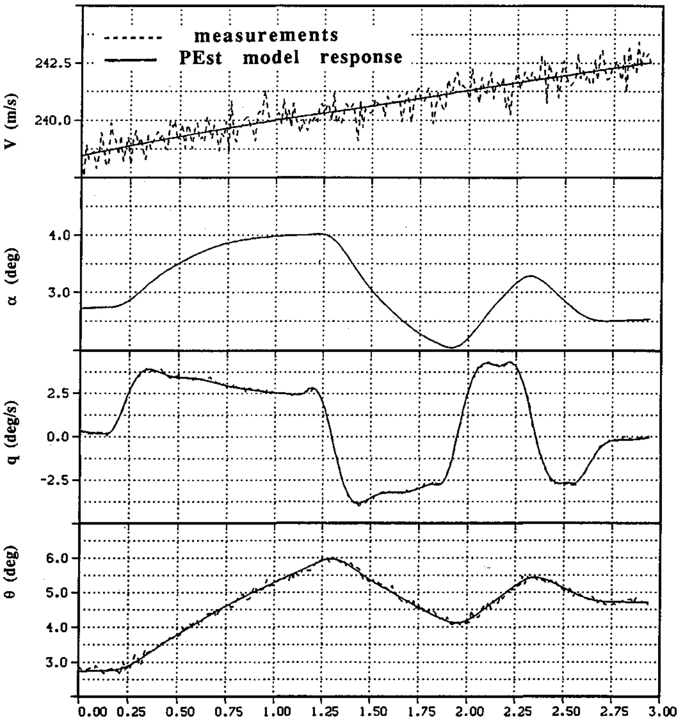

Fig. 7 Matching, using estimated derivatives (results) in estimation model (q equation decoupled).

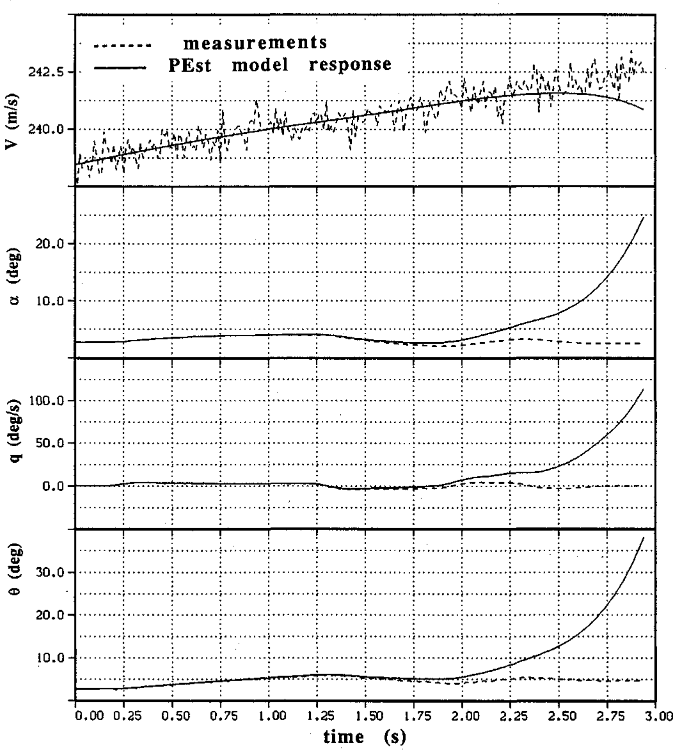

Fig. 8 Matching, using estimated derivatives (results) in estimation model (equations completely coupled).

to within 0.1%. Whereas the direct application of the output error method to this maneuver was not possible (Fig. 3), an estimation using completely decoupled equations was successful, yielding a matching of the maneuver, as shown in Fig. 5.

The α time history was calculated from the load factor and angular rate data, which is the reason for the big signal-to-noise ratio.

Table 1 shows the complete list of the derivatives that were used in the model. The derivatives that have been estimated are noted. All of the other derivatives were kept fixed at their reference values. The relative offsets of the starting values and final values of the estimated derivatives as referred to the corresponding reference data are also shown in Table 1.

The convergence behavior of the parameter estimation, including the residual biases as referred to the reference data, is shown in Fig. 6. It can be seen that after three iterations every parameter is at its final value, showing a residual bias to the reference data due to the aerodynamic linearization (model) error. In systems without model error, the bias is well below 0.01% after three iterations. A matching, using the estimated derivatives and only the q equation decoupled, is shown in Fig. 7, giving again a very good matching.

Figure 8 shows the matching, using the estimated derivatives and completely coupled equations. There is, although less distinct as in Fig. 3, the divergent behavior again. This is mainly a consequence of the aerodynamic model error always present in realistic flight test conditions, which is only partially compensated by the estimated derivatives. But referring to Fig. 7, it is stated that the estimated derivatives are adequate to describe the system for practical applications.

It is therefore concluded that the equation decoupling technique, implemented in the output error method, is an efficient tool for parameter estimation in both stable and highly unstable systems.

Concluding Remarks

The direct application of the output error method for parameter estimation purposes for highly unstable aircraft is not possible for practical applications. The equation decoupling method, implemented in an output error algorithm eliminates this shortcoming and provides an efficient tool for aerodynamic parameter estimation via the output error method in both stable and highly unstable systems.

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